THE USE OF ANALYTICAL GROUND-WATER MODELS IN MINING MONITORING

This article discusses how pocket computers can help design monitoring networks.

by William C. Walton

The design of mine monitoring networks can be enhanced through the application of analytical ground-water models. Pre-monitoring modeling efforts can provide valuable insight into probable aquifer system response to mining activities and aid in defining desirable time and space characteristics of monitoring networks.

There are several analytical models which can be applied to mine monitoring network design. Polynomial and other approximations of model equations simplify programming of pocket computers and provide rapid and inexpensive solution of model equations.

Of course, no model can be expected to answer every question. In fact, applying simple models to complex situations must be done with extreme caution.

Analytical Models for the Design of Mine Monitoring Networks

Several flow and mass transport analytical models are frequently used in designing mine monitoring networks. For example, Figure 1 shows a linear surface mine box cut or deep mine drift fully penetrating a uniformly porous model aquifer overlain and underlain by aquicludes. The nonleaky artesian aquifer is homogeneous, isotropic, infinite in areal extent and constant in thickness throughout. As the box cut or drift is excavated in the aquifer, a portion becomes unconfined and the distance to which the unconfined portion extends into the aquifer becomes larger with time.

With the box cut or drift having an infinitesimal width and therefore no storage capacity and the length of the box cut or drift increasing linearly with time, the equations governing the discharge to the box cut or drift are (McWhorter 1981):

when $t \leq \frac{L_{BD}}{E_{BD}}$

$$Q_{BD} = 4E_{BD}\left[\frac{S_yTm^2}{2} + \frac{STH_{oa}^2}{4}\right]^{1/2} t^{1/2} \qquad (1)$$

when $t \geq \frac{L_{BD}}{E_{BD}}$

$$Q_{BD} = 4E_{BD}\left[\frac{S_yTm^2}{12} + \frac{STH_{oa}^2}{4}\right]^{1/2} \qquad (2)$$

$$\left[t^{1/2} - \left(t - \frac{L_{BD}}{E_{BD}}\right)^{1/2}\right]$$

where

Q_{BD} = discharge to both sides of box cut or drift (L^3T^{-1})
E_{BD} = box cut or drift excavation rate (LT^{-1})
S_y = aquifer specific yield (dimensionless)
S = aquifer storage coefficient (dimensionless)
T = aquifer transmissivity (L^2T^{-1})
m = aquifer thickness (L)
H_{oa} = pre-mining piezometric height above top of aquifer (L)
t = time after box cut or drift excavation started (T)
L_{BD} = maximum length of box cut or drift (L)

Equations 1 and 2 apply to a water-table aquifer if H_{oa} is set equal to zero.

Discharge from a surface mine box cut, deep mine drift or room and pillar deep mine and associated drawdowns can be estimated with equivalent well-array and successive approximation techniques. Approximate well model aquifer equations, including those simulating non-leaky artesian, leaky artesian, water table and partial penetration conditions, are utilized. Rectangular arrays of wells which produce required drawdowns within specified mine areas give satisfactory approximations when the wells are closely spaced. Groups of wells may be replaced with hypothetical single wells having large radii and producing required drawdowns. Well arrays are expanded consistent with the rate of mine-area excavation.

Successive approximation techniques are used to simulate constant drawdown within mine areas and variable mine discharge. Mutual interference of individual wells is taken into consideration. The discharge of the first well in an array is selected to obtain the required drawdown within the first segment of the mine area. As wells are added to the array to simulate mine expansion, the discharges of existing and new wells are mutually adjusted to maintain required drawdown in mine segments thereby compensating for well interference. This process is continued until expansion of mine areas is completed; thereafter, the discharge of the total well array is adjusted in response to increased drawdowns with time.

The discharge of wells to obtain required drawdowns is estimated for selected time increments. Drawdowns due to estimated discharges are computed and compared with required drawdowns. If computed and required drawdowns are equal, estimated discharge is declared valid. Otherwise, discharge is re-estimated and the process is repeated until computed and required drawdowns are equal. Well model aquifer equations simulate discharge conditions when mine areas partially penetrate an aquifer.

Simulation of mine drainage may require consideration of conversion from artesian to water-table conditions and a reduction in transmissivity with dewatering. Vertical leakage may be limited to a maximum rate when hydraulic gradients reach maximum values.

Figure 2 shows two mutually leaky artesian model aquifers. With negligible aquitard storage change and assuming production and observation wells have infinitesimal diameters and no storage capacity, the equations governing the response of the aquifer system to constant pumping from a production well in aquifer 2 when the diffusivities of the two aquifers are approximately equal are (Hantush 1967):

$$s_1 = \frac{Q_2}{4\pi(T_1 + T_2)} W(u) - W(u, \beta_v) \qquad (3)$$

$$s_2 = \frac{Q_2}{4\pi(T_1 + T_2)} W(u) + \delta_1 W(u, \beta_v) \qquad (4)$$

where

$U = r^2 S_2 / 4T_2 t$

$\beta_v^2 = \beta_1^2(1 + \delta_1) = \beta_2^2(1 + \delta_2)$ (dimensionless)
$\beta_1 = r/B_1$ (dimensionless)
$\beta_2 = r/B_2$ (dimensionless)
$\delta_1 = T_1/T_2$ (dimensionless)
$\delta_2 = T_2/T_1$ (dimensionless)
$B_1 = \sqrt{T_1/(P'/m')}$ (L)
$B_2 = \sqrt{T_2/(P'/m')}$ (L)

and

T_1 = aquifer 1 transmissibility = (L^2T^{-1})
T_2 = aquifer 2 transmissibility = (L^2T^{-1})
S_1 = aquifer 1 storage coefficient (dimensionless)
S_2 = aquifer 2 storage coefficient (dimensionless)
s_1 = drawdown in aquifer 1 (L)
s_2 = drawdown in aquifer 2 (L)
Q_2 = aquifer 2 production well pumping rate (L^3T^{-1})
r = distance from production well (L)
P' = aquitard permeability (LT^{-1})
m' = aquitard thickness (L)
t = time after pumping started (T)

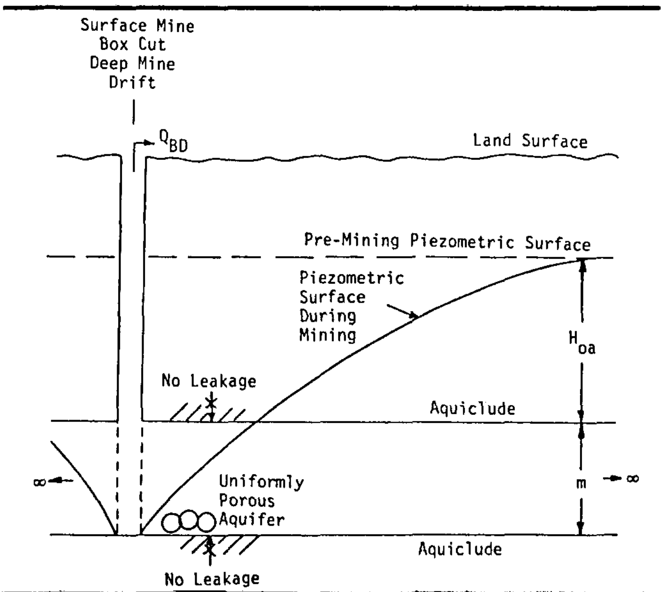

Figure 1. Uniformly porous nonleaky artesian aquifer with constant thickness and fully penetrating surface mine box cut or deep-mine drift

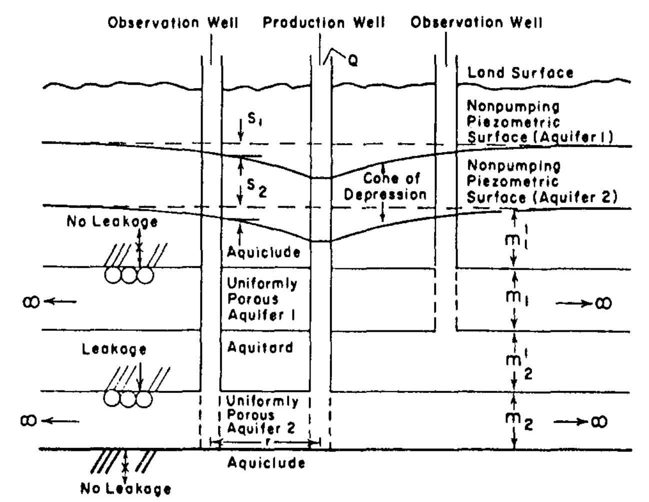

Figure 2. Uniformly porous two mutually leaky artesian aquifer system with fully penetrating wells

Figure 3 shows a solute-injection well fully penetrating a nonleaky aquifer. The density and viscosity of the injected solute are the same as those of the native water in the aquifer. A conservative solute of constant concentration is introduced into the aquifer at a constant rate through the solute-injection well. There is steady-state uniform regional flow in the aquifer parallel to the x-axis. The solute entrance rate is negligible in relation to the uniform regional flow rate. The equation governing advection and dispersion in the aquifer is (Wilson and Miller 1978):

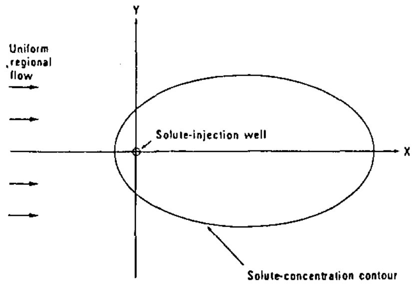

Figure 3. Uniformly porous nonleaky artesian aquifer system with solute injection

$$C_a = \frac{C_0 Q_s \exp \frac{x}{B\alpha}}{4\pi m\theta \sqrt{D_L D_T}} W_s\left(u_\alpha, \frac{r_\alpha}{B_\alpha}\right) \qquad (5)$$

where

$$u_\alpha = \frac{r_\alpha^2}{4 D_L t_{ss}} \text{ (dimensionless)} \qquad (6)$$

$$r_\alpha^2 = x^2 + y^2 (D_L/D_T) \; (L^2) \qquad (7)$$

$$B_\alpha = 2 D_L/v_s \; (L) \qquad (8)$$

$$D_L = \alpha_L v_s \; (L^2T^{-1}) \qquad (9)$$

$$D_T = \alpha_L v_s \; (L^2T^{-1}) \qquad (10)$$

and

Q_s = solute injection rate (L^3T^{-1})
C_a = change in aquifer solute concentration at solute observation well (ML^{-3})
C_o = solute injection concentration (ML^{-3})
v_s = seepage velocity of uniform regional flow (LT^{-1})
m = aquifer thickness (L)
θ = aquifer effective porosity (dimensionless)
t_{ss} = time after solute injection started (T)
D_L = aquifer longitudinal dispersion coefficient (L^2T^{-1})
D_T = aquifer transverse dispersion coefficient (L^2T^{-1})
α_L = aquifer longitudinal dispersivity (L)
α_T = aquifer transverse dispersivity (L)
x,y = coordinates (L)

Equation 5 also describes advection and dispersion from a point source of contamination assuming contaminants are distributed instantaneously to the entire aquifer thickness beneath the point source. Advection and dispersion from a source area may be simulated with an array of closely spaced point sources. The source area and solute injection strength are divided into equal parts and assigned to point sources. The cumulative effects of individual point sources are then determined by the method of superposition. The point source assumption results in errors of less than 10 percent when distances to observation wells exceed

$$.2\alpha_L \left(1 = \frac{\alpha_L}{\alpha_T}\right) \left[\frac{\text{source area represented}}{\text{source length or width } (\alpha_L)}\right]^2.$$

Approximations of Well Functions in Analytical Model Equations

Programming pocket computers depends upon algebraic polynomial and other approximations of the complex well functions in analytical model equations. Polynomial approximations for many well functions may be found in National Bureau of Standards (1964) and Abramowitz and Stegun (1970). Programming may involve solutions of algebraic equations by Newton's method, integral estimation by Gauss-LeGendre three-point quadrature, two-point Laguerre integration technique and incremental curve fitting techniques (Abramowitz and Stegun 1970). Problems involving models with boundaries and multiple-well systems may be solved utilizing the x-y coordinate pocket computers and summations of drawdowns and buildups due to real and image wells. Polynomial and other approximations of well functions involved in frequently used model equations are:

$W(u, \frac{r}{B})$

When $\frac{r}{B} < 2$ and $u > 1$

$$W(u,\frac{r}{B}) = I_o(\frac{r}{B})W(u) - \exp(-u)\left[0.5772 + \ln(r^2/4B^2u) + W(r^2/4B^2u) - (r^2/4B^2u) + \frac{I_o(\frac{r}{B}) - 1}{u}\right] + \exp(-1/u) \sum_{n=1}^{6} \sum_{m=1}^{n} \frac{(-1)^{n+m}(n-m+1)!}{(n+2)!^2\, u^{n-m}} \left(\frac{r^2}{4b^2}\right)^n$$

When $\frac{r}{B} \leq 2$ and $u \leq 1$

$$W(u,\frac{r}{B}) = 2K_o(\frac{r}{B}) - \left[I_o(\frac{r}{B})W(r^2/4B^2u) - \exp(r^2/4B^2u)\left[0.5772 + \ln(u) + W(u) - u + \frac{I_o(\frac{r}{B}) - 1}{r^2/4B^2u}\right] + u^2 \sum_{n=1}^{6} \sum_{m=1}^{n} \frac{(-1)^{n+m}(n-m+1)!}{(n+2)!^2 u^{m-n}} \left(\frac{r^2}{4B^2}\right) m\right]$$

When $\frac{r}{B} > 2$

$$W(u,\frac{r}{B}) = \left(\frac{\pi B}{2r}\right)^{1/2} \exp(-\frac{r}{B})\, \text{erfc}\left[-\frac{\frac{r}{B} - 2u}{2u^{1/2}}\right]$$

$W(\frac{r}{m}\sqrt{P_v/P_h}, \frac{1}{m}, \frac{d}{m}, \frac{y}{m})$

$$W(\frac{r}{m}\sqrt{P_v/P_h}, \frac{1}{m}, \frac{d}{m}, \frac{y}{m}) = [4m/\pi(1-d)] \sum_{n=1}^{\infty} (1/n)\,[\sin(n\pi 1/m) - \sin(n\pi d/m)] \cos(n\pi y/m)\, K_o\,[n\pi r/m)(P_v/P_h)^{1/2}]$$

W(u)

When $0 < u \leq 1$

$$W(u) = -\ln u + a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5$$

$a_0 = -.57721566$
$a_1 = .99999193$
$a_2 = -.24991055$
$a_3 = .05519968$
$a_4 = -.00976004$
$a_5 = .00107857$

When $1 \leqq u < \infty$

$$W(u) = \left[\frac{u^4 + a_1 u^3 + a_2 u^2 + a_3 u + a_4}{u^4 + b_1 u^3 + b_2 u^2 + b_3 u + b_4}\right] / u \exp(u)$$

$a_1 = 8.5733287401$
$a_2 = 18.0590169730$
$a_3 = 8.6347608925$
$a_4 = .2677737343$
$b_1 = 9.5733223454$
$b_2 = 25.6329561486$
$b_3 = 21.0996530827$
$b_4 = 3.9584969228$

$K_o(x)$

When $0 < x \leqq 2$

$$K_o(x) = -\ln(x/2)I_o(x) - .57721566 + .42278420\ (x/2)^2 + .23069756\ (x/2)^4 + .03488590\ (x/2)^6 + .00262698\ (x/2)^8 + .00010750\ (x/2)^{10} + .00000740\ (x/2)^{12}$$

When $2 \leqq x < \infty$

$$K_o(x) = [1.25331414 - .07832358\ (2/x) + .02189568\ (2/x)^2 - .01062446\ (2/x)^3 + .00587872\ (2/x)^4 - .00251540\ (2/x)^5 + .00053208\ (2/x)^6] / x^{1/2} \exp(x)$$

$I_o(x)$

When $-3.75 \leqq x \leqq 3.75$

$$I_o(x) = 1 + 3.5156229\ (x/3.75)^2 + 3.0899424\ (x/3.75)^4 + 1.2067492\ (x/3.75)^6 + .2659732\ (x/3.75)^8 + .0360768\ (x/3.75)^{10} + .0045813\ (x/3.75)^{12}$$

erf(x)

$$\mathrm{erf}(x) = 1 - 1/(1 + a_1 x + a_2 x^2 + \ldots a_6 x^6)^{16}$$

$a_1 = .0705230784$
$a_2 = .0422820123$
$a_3 = .0092705272$
$a_4 = .0001520143$
$a_5 = .0002765672$
$a_6 = .0000430638$

erfc(x)

$$\mathrm{erfc}(x) = 1/(1 + a_1 x + a_2 x^2 + \ldots a_6 x^6)^{16}$$

The following relations are useful:

$\mathrm{erf}(-x) = -\mathrm{erf}(x)$, $\mathrm{erf}(o) = 0$, $\mathrm{erf}(\infty) = 1$,
$\mathrm{erfc}(-x) = 1 + \mathrm{erf}(x)$, $\mathrm{erfc}(-\infty) = 2$

for $x < 0.1$, $\mathrm{erf}(x) \cong 2x/\sqrt{\pi}$
for $x > 9$, $\exp(x)\ \mathrm{erfc}(\sqrt{x}) \cong 1/\sqrt{\pi x}$

```
10:BEEP 3:PRINT
   "NON-LEAKY A
   RTESIAN, 1 W
   ELL":BEEP 1
20:INPUT "RADIU
   S(FT)=";R:
   BEEP 1
25:PRINT "RADIU
   S(FT)=";
   USING "##.##
   ##^";R
30:INPUT "T(GPD
   /FT)=";T:
   BEEP 1
35:PRINT "T(GPD
   /FT)=";USING
   "##.####^";T
40:INPUT "STORA
   GE COEF.=";S
   :BEEP 1
45:PRINT "STORA
   GE COEF.=";
   USING "##.##
   ##^";S
50:INPUT "PUMPI
   NG RATE(GPM)
   =";Q:BEEP 1
55:PRINT "PUMPI
   NG RATE(GPM)
   =";USING "##
   .####^";Q
60:INPUT "TIME(
   MIN)=";Z:
   BEEP 1
65:PRINT "TIME(
   MIN)=";USING
   "##.####^";Z
70:U=2693*S*R^2
   /(T*Z):BEEP
   1
80:PRINT "U=";
   USING "##.##
   ##^";U
90:A=U^2:B=U^3:
   C=U^4:D=U^5
100:IF U>1GOTO 1
    30
110:W=-LN U-.577
    21566+.99999
    193*U-.24991
    055*A+.05519
    968*B-.00976
    004*C+.00107
    857*D
120:GOTO 160
130:M=C+8.573328
    7401*B+18.05
    90169730*A+8
    .634760825*
    U+.267773734
    3
140:N=M/(C+9.573
    322454*B+25
    .632956148 6*
    A+21.0965308
    27*U+3.95849
    69228)
150:W=N(U*EXP U)
160:BEEP 2:PRINT
    "W(U)=";
    USING "##.##
    ##^";W
170:H=114.6*Q*W/
    T:BEEP 2
180:PRINT "DRAWD
    OWN(FT)=";
    USING "##.##
    ##^";H
190:END
```

```
NON-LEAKY ARTESI
AN, 1 WELL
RADIUS(FT)= 1.00
00E 02
T(GPD/FT)= 1.000
0E 05
STORAGE COEF.= 4
.0000E-04
PUMPING RATE(GPM
)= 1.0000E 02
TIME(MIN)= 3.000
0E 02
U= 3.5906E-04
W(U)= 7.3551E 00
DRAWDOWN(FT)= 8.
4289E-01
```

Pocket Computer Program Codes

Example program codes involving flow and mass transport in a nonleaky artesian aquifer system and developed for the Radio Shack TRS-80 pocket computer and associated Printer/Cassette Interface are presented here. The TRS-80 programs are written in BASIC language. The TRS-80 has a program capacity of 250 lines, 1,424 bytes of user-addressable memory and 26 memories with memory safeguard. A cassette recorder tape stores programs.

References

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Biographical Sketch

William C. Walton manages the Geraghty & Miller Midwest office and is responsible for organizing and directing the firm's hydrogeologic investigations in that area of the country. Prior to joining Geraghty & Miller, he served as executive director of the Upper Mississippi River Basin Commission. He directed the Water Research Center at the University of Minnesota and lectured on water resources management. Prior to these executive responsibilities, Walton worked as a hydraulic and civil engineer for the Illinois State Water Survey, the U.S. Geological Survey and the Bureau of Reclamation. He was the first editor of Ground Water and he has published more than 75 technical journal articles. He is author of two books, Ground Water Resource Evaluation and World of Water.